



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Let $\cos ax/(1+x^{2n})=f(x)$.

$$\begin{aligned}\int_{-\infty}^{\infty} f(x)dx &= \int_0^{\infty} f(x)dx + \int_{-\infty}^0 f(x)dx = \int_0^{\infty} f(x)dx - \int_{\infty}^0 f(x)dx = \int_0^{\infty} f(x)dx \\ &+ \int_0^{\infty} f(x)dx = 2 \int_0^{\infty} f(x)dx.\end{aligned}$$

$$\therefore \int_0^{\infty} f(x)dx = \frac{1}{2} \int_{-\infty}^{\infty} f(x)dx = -i \frac{\pi}{2n} \sum_{r=1}^n \omega^{2r-1} e^{a i \omega^{2r-1}} \omega^{2r-1} .]$$

71. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Form the differential equation of the third order, of which

$y=c_1 e^{2x}+c_2 e^{-3x}+c_3 e^x$ is the complete primitive.

I. Solution by EDGAR ODELL LOVETT, Ph. D., Princeton University, Princeton, N. J.

1°. This problem is a familiar one to students of differential equations. The original primitive together with the results of three successive differentiations, may be written

$$\begin{aligned}y - e^{2x}c_1 - e^{-3x}c_2 - e^x c_3 &= 0, \\ y' - 2e^{2x}c_1 + 3e^{-3x}c_2 - e^x c_3 &= 0, \\ y'' - 4e^{2x}c_1 - 9e^{-3x}c_2 - e^x c_3 &= 0, \\ y''' - 8e^{2x}c_1 + 27e^{-3x}c_2 - e^x c_3 &= 0;\end{aligned}$$

where $y' \equiv \frac{dy}{dx}$, $y'' \equiv \frac{d^2 y}{dx^2}$, $y''' \equiv \frac{d^3 y}{dx^3}$.

The above is a system of linear and homogeneous equations in the quantities 1, $e^{2x}c_1$, $e^{-3x}c_2$, and $e^x c_3$, hence the determinant of their coefficients vanishes, that is

$$\begin{vmatrix} y & 1 & 1 & 1 \\ y' & 2 & -3 & 1 \\ y'' & 4 & 9 & 1 \\ y''' & 8 & -27 & 1 \end{vmatrix} \equiv \begin{vmatrix} y & 1 & 1 & 1 \\ y'-y & 1 & -4 & 0 \\ y''-y & 3 & 8 & 0 \\ y'''-y & 7 & -28 & 0 \end{vmatrix} \equiv 4 \begin{vmatrix} y'-y & 1 & 1 \\ y''-y & 3 & -2 \\ y'''-y & 7 & 7 \end{vmatrix} \equiv 0;$$

whence

$$\begin{vmatrix} y'-y & -1 & 0 \\ y''-y & 3 & -2 \\ y'''-y & 7 & 0 \end{vmatrix} \equiv 2 \begin{vmatrix} y'-y & 1 \\ y'''-y & 7 \end{vmatrix} = 0;$$

or finally

$$y''' - 7y' + 6y = 0$$

is the differential equation of the third order whose complete primitive is

$$y - ae^{2x} - be^{-3x} - ce^x = 0.$$

2°. If the problem be generalized and the complete primitive taken in the form

$$y - ae^{mx} - be^{-(m+n)x} - ce^{nx} = 0,$$

the corresponding differential equation of the third order is readily found to be

$$y''' - (m^2 + mn + n^2)y' + mn(m+n)y = 0.$$

The values $m=2$ and $n=1$ give the original problem.

3°. If the problem be completely generalized and the original primitive taken in the form

$$y - ae^{px} - be^{qx} - ce^{rx} = 0,$$

the differential equation is

$$y''' - (p+q+r)y'' + (pq+qr+rp)y' - pqry = 0.$$

Putting $p+q+r=0$ we have the second case above. If in addition to $p+q+r=0$, $p=2$ and $q=1$, the first particular case appears again.

II. Solution by **WALTER HUGH DRANE, A. M.**, Professor of Mathematics, Jefferson Military College, Washington, Miss.

(1) $y = c_1 e^{2x} + c_2 e^{-3x} + c_3 e^x$. Differentiate (1).

(2) $\frac{dy}{dx} = 2c_1 e^{2x} - 3c_2 e^{-3x} + c_3 e^x$. Subtract (1) from (2).

(3) $\frac{dy}{dx} - y = c_1 e^{2x} - 4c_2 e^{-3x}$. Differentiate (3).

(4) $\frac{d^2 y}{dx^2} - \frac{dy}{dx} = 2c_1 e^{2x} + 12c_2 e^{-3x}$. Subtract twice (3) from (4).

(5) $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = 20c_2 e^{-3x}$. Differentiate (5).

(6) $\frac{d^3 y}{dx^3} - 3\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} = -60c_2 e^{-3x}$. Add 3 times (5) to (6).

(7) $\frac{d^3 y}{dx^3} - 7\frac{dy}{dx} + 6y = 0$. Q. E. D.

See Johnson's Differential Equations, page 104, example 7.

MECHANICS.

61. Proposed by **WILLIAM HOOVER, A. M.**, Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

A body is suspended from a fixed point by an elastic string, which is stretched to double its natural length when the body is in equilibrium. Find how much the body must be depressed, so that when let go, it may just reach the point of suspension.